Chapter 7

A General Strategy for Solving Material Balance Problems

7.1 Problem Solving

An orderly method of analyzing problems and presenting their solutions represents training in logical thinking that is of considerably greater value than mere knowledge of how to solve a particular type of problem.

7.2 The Strategy for Solving Problems

1. Read and understand the problem statement.
2. Draw a sketch of the process and specify the system boundary.
3. Place labels for unknown variables and values for known variables on the sketch.
4. Obtain any missing needed data.
5. Choose a basis.
6. Determine the number of unknowns.
7. Determine the number of independent equations, and carry out a degree of freedom analysis.
8. Write down the equations to be solved.
9. Solve the equations and calculate the quantities asked for.
10. Check your answer.

Example 7.1

A thickener in a waste disposal unit of a plant removes water from wet sewage sludge as shown in Figure E7.l. How many kilograms of water leave the thickener per 100 kg of wet sludge that enter the thickener? The process is in the steady state.

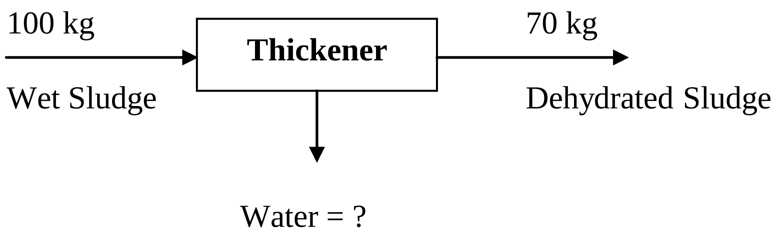


Figure E7.1

Solution

Basis: 100 kg wet sludge

The system is the thickener (an open system). No accumulation, generation, or consumption occurs. The total mass balance is

$$\underline{\text{In}} \quad = \quad \underline{\text{Out}}$$

$$100 \text{ kg} = 70 \text{ kg} + \text{kg of water}$$

Consequently, the water amounts to 30 kg.

Example 7.2

A continuous mixer mixes NaOH with H_2O to produce an aqueous solution of NaOH. Determine the composition and flow rate of the product if the flow rate of NaOH is 1000 kg/hr, and the ratio of the flow rate of the H_2O to the product solution is 0.9. For this process,

1. Sketch of the process is required.
2. Place the known information on the diagram of the process.
3. What basis would you choose for the problem?
4. How many unknowns exist?
5. Determine the number of independent equations.
6. Write the equations to be solved.
7. Solve the equations.
8. Check your answer.

Solution

1. The process is an open one, and we assume it to be steady state.

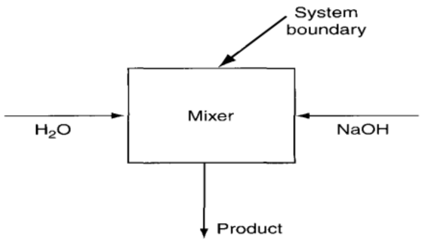
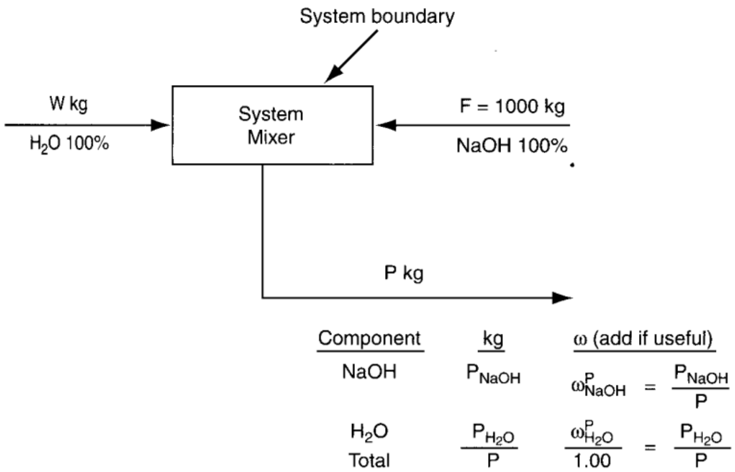


Figure E7.2

2. Because no contrary information is provided about the composition of the H_2O and NaOH streams, we will assume that they are 100% H_2O and NaOH, respectively.



3. Basis (1000 kg or 1 hour or 1000 kg/hr) (all are equivalent)

4. We do not know the values of four variables: W, P, P_{NaOH} and P_{H2O}.

5. You can write three material balances:

- one for the NaOH
- one for the H_2O
- one total balance (the sum of the two component balances)

Only two are independent.

<u>Note:</u> You can write as many independent material balances as there are species involved in the system.

6. Material balance: in = out or in – out = 0

$$\text{NaOH balance:} \quad 1000 = P_{NaOH} \quad \text{or} \quad 1000 - P_{NaOH} = 0 \qquad (1)$$

$$H_2O \text{ balance:} \quad W = P_{H_2O} \quad \text{or} \quad W - P_{H_2O} = 0 \qquad (2)$$

$$\text{Given ratio:} \quad W = 0.9P \quad \text{or} \quad W - 0.9P = 0 \qquad (3)$$

$$\text{Sum of components in } P\text{: } P_{NaOH} + P_{H_2O} = P \text{ or } P_{NaOH} + P_{H_2O} - P = 0 \qquad (4)$$

Could you substitute the total mass balance 1000 + W = P for one of the two component mass balances? Of course In fact, you could calculate P by solving just two equations:

$$\text{Total balance:} \quad 1000 + W = P$$

$$\text{Given ratio:} \quad W = 0.9P$$

7. Solve equations:

W = 0.9 P substitute in total balance 1000 + 0.9 P = P

∴ P = 10000 kg & W = 0.9 * 10000 = 9000 kg (The basis is still 1 hr (F_{NaOH} = 1000 kg))

From these two values you can calculate the amount of H_2O and NaOH in the product

$$\text{From the } \begin{cases} \text{NaOH balance:} \\ H_2O \text{ balance:} \end{cases} \text{you get } \begin{cases} P_{NaOH} = 1000 \text{ kg} \\ P_{H_2O} = 9000 \text{ kg} \end{cases}$$

Then

$$\omega^P_{NaOH} = \frac{1000 \text{ kg NaOH}}{10{,}000 \text{ kg Total}} = 0.1$$

$$\omega^P_{H_2O} = \frac{9{,}000 \text{ kg } H_2O}{10{,}000 \text{ kg Total}} = 0.9$$

<u>Note</u>

$$\omega^P_{NaOH} + \omega^P_{H_2O} = 1$$

8. The total balance would have been a redundant balance, and could be used to check the answers

$$P_{NaOH} + P_{H2O} = \dot{P}$$

1,000 + 9,000 = 10,000

<u>Note:</u> After solving a problem, use a <u>redundant equation</u> to check your values.

Degree of Freedom Analysis

The phrase degrees of freedom have evolved from the design of plants in which fewer independent equations than unknowns exist. The difference is called the degrees of freedom available to the designer to specify flow rates, equipment sizes, and so on. You calculate the number of degrees of freedom (N_D) as follows:

Degrees of freedom = number of unknowns — number of independent equations

$$N_D = N_U - N_E$$

* When you calculate the number of degrees of freedom you ascertain the solve ability of a problem. **Three** outcomes exist:

Case	N_D	Possibility of Solution
$N_U = N_E$	0	Exactly specified (determined); a solution exists
$N_U > N_E$	>0	Under specified (determined); more independent equations required
$N_U < N_E$	<0	Over specified (determined)

For the problem in **Example 7.2**,

$N_U = 4$

$N_E = 4$

So that

$N_D = N_U - N_E = 4 - 4 = 0$

And a **unique** solution exists for the problem.

Example 7.3

A cylinder containing CH_4, C_2H_6, and N_2 has to be prepared containing a CH_4 to C_2H_6 mole ratio of 1.5 to 1. Available to prepare the mixture is (l) a cylinder containing a mixture of 80% N_2 and 20% CH_4, (2) a cylinder containing a mixture of 90% N_2 and 10% C_2H_6, and (3) a cylinder containing pure N_2. What is the number of degrees of freedom, i.e., the number of independent specifications that must be made, so that you can determine the respective contributions from each cylinder to get the desired composition in the cylinder with the three components?

Solution

A sketch of the process greatly helps in the analysis of the degrees of freedom. Look at Figure E7.3.

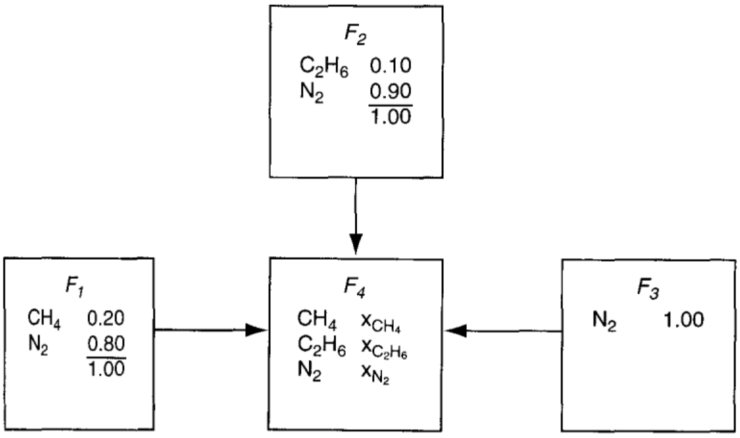


Figure E7.3

Do you count **seven unknowns** — **three** values of $\mathbf{x_i}$ and **four** values of $\mathbf{F_i}$? How many independent equations can be written?

- Three material balances: CH_4, C_2H_6, and N_2
- One specified ratio: moles of CH_4 to C_2H_6 equal 1.5 or $(X_{CH4}/X_{C2H6}) = 1.5$
- One summation of mole fractions: $\sum x_i^{F_4} = 1$

Thus, there are **seven minus five equals two degrees of freedom** ($N_D = N_U - N_E = 7 - 5 = 2$). If you pick a basis, such as $F_4 = 1$, one other value has to be specified to solve the problem to calculate composition of F_4.

Questions

1. What does the concept "solution of a material balance problem" mean?
2. (a) How many values of unknown variables can you compute from one independent material balance?

   (b) From three independent material balance equations?

   (c) From four material balances, three of which are independent?
3. If you want to solve a set of independent equations that contain fewer unknown variables than equations (the over specified problem), how should you proceed with the solution?
4. What is the major category of implicit constraints (equations) you encounter in material balance problems?
5. If you want to solve a set of independent equations that contain more unknown variable than equations (the underspecified problem), what must you do to proceed with the solution?

Answers:

1. A solution means a (possibly unique) set of values for the unknowns in a problem that satisfies the equations formulated in the problem.
2. (a) one; (b) three; (c) three.
3. Delete nonpertinent equations, or find additional variables not included in the analysis.
4. The sum of the mass or mole fraction in a stream or inside a system is unity.
5. Obtain more equations or specifications, or delete variables of negligible importance.

Problems

1. A water solution containing 10% acetic acid is added to a water solution containing 30% acetic acid flowing at the rate of 20 kg/min. The product P of the combination leaves the rate of 100 kg/min. What is the composition of P? For this process,
    a. Determine how many independent balances can be written.
    b. List the names of the balances.
    c. Determine how many unknown variables can be solved for.
    d. List their names and symbols.
    e. Determine the composition of P.
2. Can you solve these three material balances for F, D, and P? Explain why not.

$$0.1F + 0.3D = 0.2P$$
$$0.9F + 0.7D = 0.8P$$
$$F + D = P$$

3. How many values of the concentrations and flow rates in the process shown in Figure SAT7.2P3 are unknown? List them. The streams contain two components, 1 and 2.

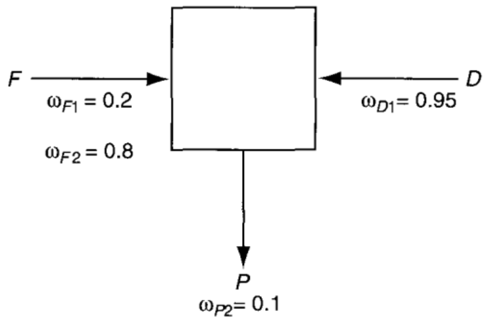


Figure SAT7.2P3

4. How many material balances are needed to solve problem 3? Is the number the same as the number of unknown variables? Explain.

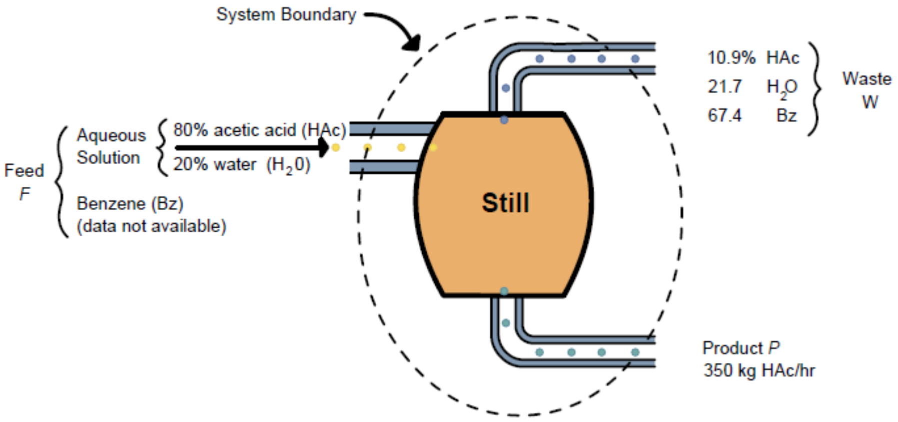
Answers:

1. (a) Two; (b) two of these three: acetic acid, water, total; (c) two; (d) feed of the 10% solution (say F) and mass fraction ω of the acetic acid in P; (e) 14% acetic acid and 86% water
2. Not for a unique solution because only two of the equations are independent.
3. F, D, P, ω_{D2}, ω_{P1}
4. Three unknowns exist. Because only two independent material balances can be written for the problem, one value of F, D, or P must be specified to obtain a solution. Note that specifying values of ω_{D2} or ω_{P1} will not help.

Supplementary Problems (Chapter Seven):

Problem 1

A continuous still is to be used to separate acetic acid, water, and benzene from each other. On a trial run, the calculated data were as shown in the figure. Data recording the benzene composition of the feed were not taken because of an instrument defect. The problem is to calculate the benzene flow in the feed per hour. How many independent material balance equations can be formulated for this problem? How many variables whose values are unknown exist in the problem?



Solution

Three components exist in the problem, hence three mass balances can be written down (the units are kg):

Balance	F in		W out		P out	
HAc:	$0.80(1-\omega_{Bz,F})F$	=	0.109W	+	350	(a)
H_2O:	$0.20(1-\omega_{Bz,F})F$	=	0.217W	+	0	(b)
Benzene:	$\omega_{Bz,F}F$	=	0.67W	+	0	(c)

The total balance would be: F = W + 350 (in kg).

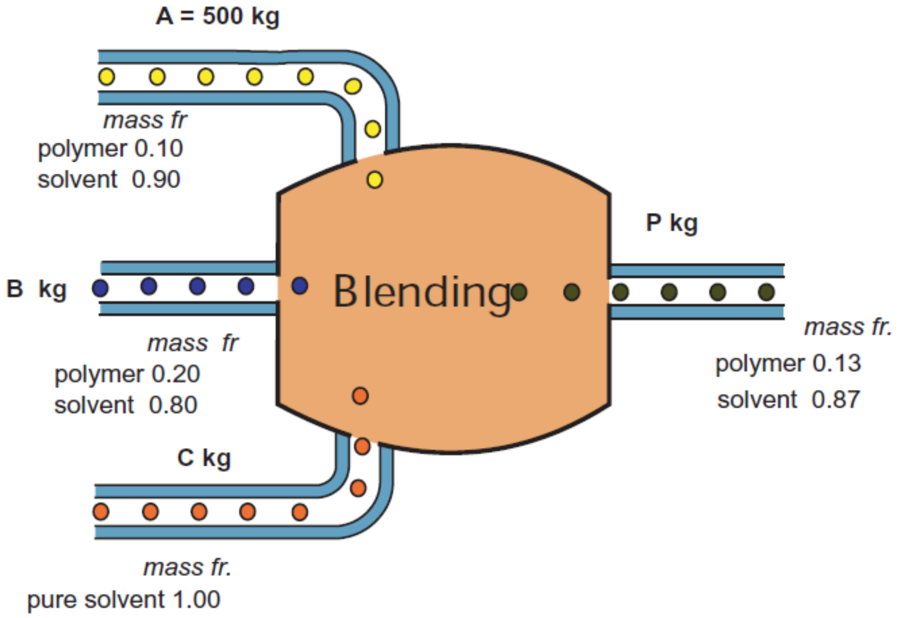
Problem 2

A liquid adhesive, which is used to make laminated boards, consists of a polymer dissolved in a solvent. The amount of polymer in the solution has to be carefully controlled for this application. When the supplier of the adhesive receives an order for 3000 kg of an adhesive solution containing 13 wt % polymer, all it has on hand is (1) 500 kg of a 10 wt % solution, (2) a very large quantity of a 20 wt % solution, and (3) pure solvent.

Calculate the weight of each of the three stocks that must be blended together to fill the order. Use all of the 10 wt % solution.

Solution

This is a steady state process without reaction.



Basis: 3000 kg 13 wt % polymer solution

Two unknowns: B and C . (A is not an unknown since all of it must be used).

Total balance: 500 + B + C = 3000 (1)

Polymer balance: 0.10 (500) + 0.20 B + 0.00 (C) = 0.13 (3000) (2)

Solvent balance: 0.90 (500) + 0.80 B + 1.00 (C) = 0.87 (3000) (3)

We will use equations (1) and (2).

from (2) 0.1 (500) + 0.20 B = 0.13 (3000) **B = 1700 kg**

from (1) 500 + 1700 + C = 3000 **C = Ê800 kg**

Equation (3) can be used as a check,

0.90 A + 0.80 B + C = 0.87 P

0.90 (500) + 0.80 (1700) + 800 = 2610 = 0.87 (3000) = 2610